Fusion Neutronics Meeting 2025

Implicit Stochastic Uncertainty Propagation Scheme for Two-Step Monte Carlo Simulations

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Two-Step MC Simulations

- Not enough statistical convergence in a single MC simulation.
- Events which involve two sequentially coupled particle sources.

First MC Simulation $\{\phi_i\}$ → Intermediate Radiation Source $I_j(\{\phi_i\})$ → Second MC Simulation D

Stochastic Uncertainty in Two-Step MC Simulations

First MC Simulation → Intermediate Radiation Source → Second MC Simulation

First MC Simulation	Intermediate Radiation Source	Second MC Simulation
↓	↓	↓
stochastic uncertainty due to MC1	deterministic calculation no stochastic uncertainty	stochastic uncertainty due to MC2

stochastic uncertainty in the final nuclear response due to MC1

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Implicit Stochastic Uncertainty Propagation Scheme

Random Variable (F) - Requirements

- Dependence only on the MC1 radiation field. ➡ $F = F(\{\phi_i\})$

- Alignment with the stochastic uncertainty in the final nuclear response due to MC1.

➡ $\mathrm{D} = \sum_j I_j R_j = \sum_j \sum_i I_j(\{\phi_i\}) R_j$

applying the uncertainty propagation law

$$\sigma^2_{\mathrm{D}_{\mathrm{MC1}}} = \sum_i \sum_{i'} \frac{\partial \mathrm{D}}{\partial \phi_i} \frac{\partial \mathrm{D}}{\partial \phi_{i'}} cov(\phi_i, \phi_{i'})$$

Random Variable (F) - Definition

$$F(\{\phi_i\}) = \sum_i \phi_i \times \frac{\partial \mathrm{D}}{\partial \phi_i} = \sum_i \phi_i \times \sum_j R_j \frac{\partial I_j}{\partial \phi_i}$$

Random Variable (F) - Requirements

$$F(\{\phi_i\}) = \sum_i \phi_i \times \frac{\partial \mathrm{D}}{\partial \phi_i} = \sum_i \phi_i \times \sum_j R_j \frac{\partial I_j}{\partial \phi_i}$$

dependence on the
MC1 radiation field

Random Variable (F) - Requirements

$$F(\{\phi_i\}) = \sum_i \phi_i \times \frac{\partial \mathrm{D}}{\partial \phi_i} = \sum_i \phi_i \times \sum_j R_j \frac{\partial I_j}{\partial \phi_i}$$

applying the uncertainty propagation law

$$\sigma_F^2 = \sum_i \sum_{i'} \frac{\partial \mathrm{D}}{\partial \phi_i} \frac{\partial \mathrm{D}}{\partial \phi_{i'}} cov(\phi_i, \phi_{i'})$$

$$\sigma^2_{\mathrm{D}_{\mathrm{MC1}}} = \sum_i \sum_{i'} \frac{\partial \mathrm{D}}{\partial \phi_i} \frac{\partial \mathrm{D}}{\partial \phi_{i'}} cov(\phi_i, \phi_{i'})$$

alignment with the stochastic uncertainty in the final nuclear response due to MC1

Random Variable (F) - Definition

$$F(\{\phi_i\}) = \sum_i \phi_i \times \frac{\partial \mathrm{D}}{\partial \phi_i} = \sum_i \phi_i \times \sum_j R_j \frac{\partial I_j}{\partial \phi_i}$$

Random Variable (F) - Dependence

$$F(\{\phi_i\}) = \sum_i \phi_i \times \boxed{\frac{\partial D}{\partial \phi_i}} = \sum_i \phi_i \times \boxed{\sum_j R_j \frac{\partial I_j}{\partial \phi_i}}$$

sensitivity coefficients

constant with respect to the MC1 radiation field

also appear when applying the uncertainty propagation law

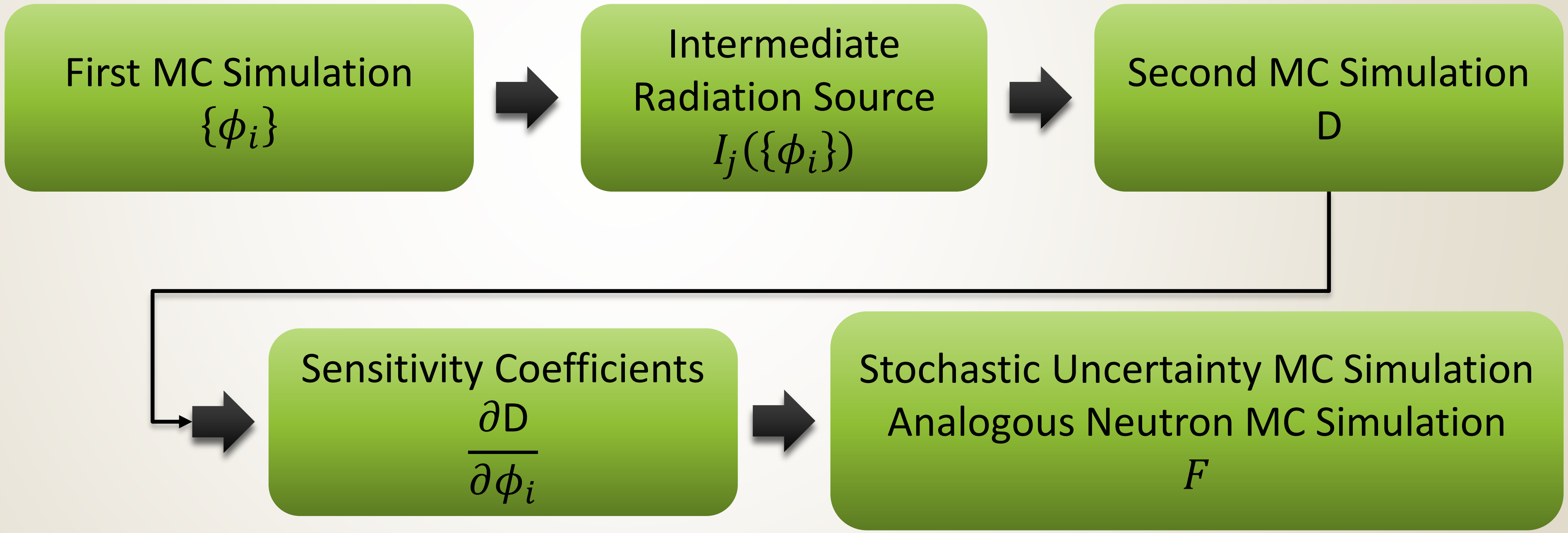
Random Variable (F) - Dependence

$$F(\boxed{\{\phi_i\}}) = \sum_i \boxed{\phi_i} \times \frac{\partial \mathrm{D}}{\partial \phi_i} = \sum_i \boxed{\phi_i} \times \sum_j R_j \frac{\partial I_j}{\partial \phi_i}$$

particles simulated during MC1

Random Variable (F) - Assessment

$$F(\{\phi_i\}) = \sum_i \phi_i \times \frac{\partial D}{\partial \phi_i}$$



sensitivity coefficients

particles simulated during MC1

Implicit Scheme Advantages

- Quantifies the stochastic uncertainty in the final nuclear response due to MC1 as the relative error of the random variable.
- MC method capabilities, such as variance reduction techniques and statistical tests.
- Eliminates the explicit calculation of the MC1 radiation field covariance matrix.
- Avoids any additional assumptions beyond those of the two-step MC simulation method.

Implicit Scheme Limitations

- Estimation of the sensitivity coefficients, no further than previous stochastic uncertainty propagation techniques.
- Modifications of MCNP, which undergoes rigorous testing.
- Stochastic uncertainty only for a predefined set of tallies.

Implementation in SRC-UNED

SRC-UNED

J. Catalan, *et al.*, “Development of radiation sources for nuclear analysis beyond ITER bio-shield: SRC-UNED code”, *Computer Physics Communications*, vol. 275, p. 108309, 2022

Couples two MC simulations using a common geometrical interface, significantly improving the statistical convergence beyond the common geometrical interface, developed within **MCUNED+** environment.

First MC Simulation → Intermediate Radiation Source → Second MC Simulation

$$F(\{\phi_i\}) = \sum_i \phi_i \times \frac{\partial D}{\partial \phi_i}$$

WSSA

=

particles simulated during MC1

source contribution to the final nuclear response

=

sensitivity coefficients

stochastic uncertainty in the final nuclear response due to MC1

SRC-UNED

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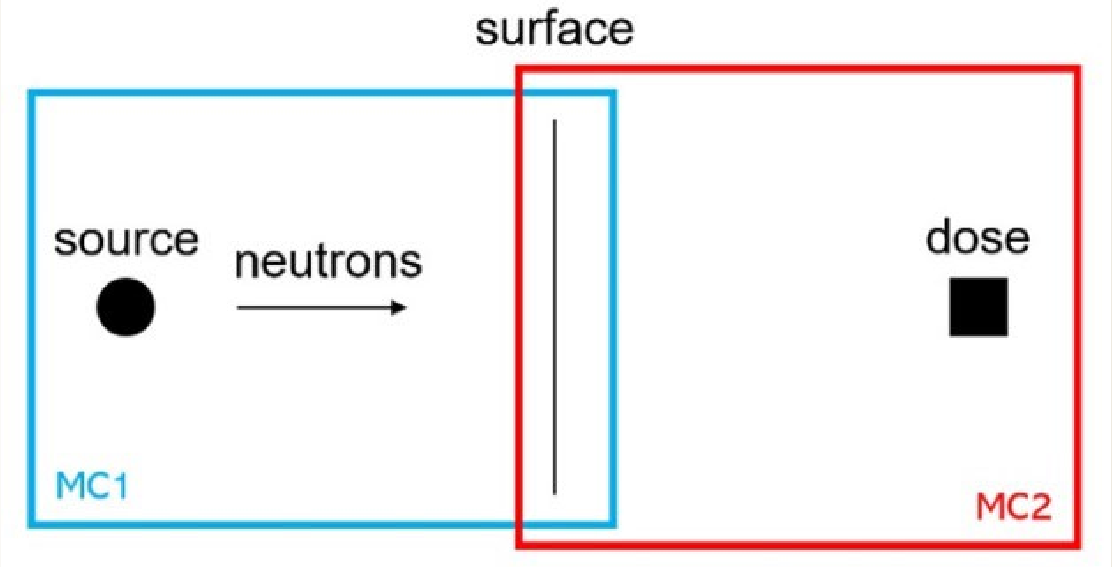

	D	$\sigma_{D_{MC1}}$	$\sigma_{D_{MC2}}$	σ_D
Analytical	50.00	2.83×10^{-2}	1.15×10^{-2}	3.05×10^{-2}
Brute Force	50.00	–	1.15×10^{-2}	3.00×10^{-2}
Implicit Scheme	49.90	2.76×10^{-2}	1.15×10^{-2}	2.99×10^{-2}

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Implementation in R2S-UNED
- JET -

R2S-UNED

P. Sauvan, *et al.*, "Development of the R2SUNED Code System for Shutdown Dose Rate Calculations", *IEEE Transactions on Nuclear Science*, vol. 63, nº 1, pp. 375-384, 2016

R2S methodology for Shutdown Dose Rate calculations developed within **MCUNED+** and **ACAB-Loop** environments.

Neutron MC Simulation
Neutron Flux
→
Activation
DGS
→
Photon MC Simulation
SDR

$$F(\{\phi_i\}) = \sum_i \phi_i \times \frac{\partial \mathrm{SDR}}{\partial \phi_i}$$

stochastic uncertainty MC simulation analogous neutron MC simulation

=

particles simulated during MC1

→ stochastic uncertainty in SDR due to MC1

source contribution to the SDR

=

sensitivity coefficients

R2S-UNED

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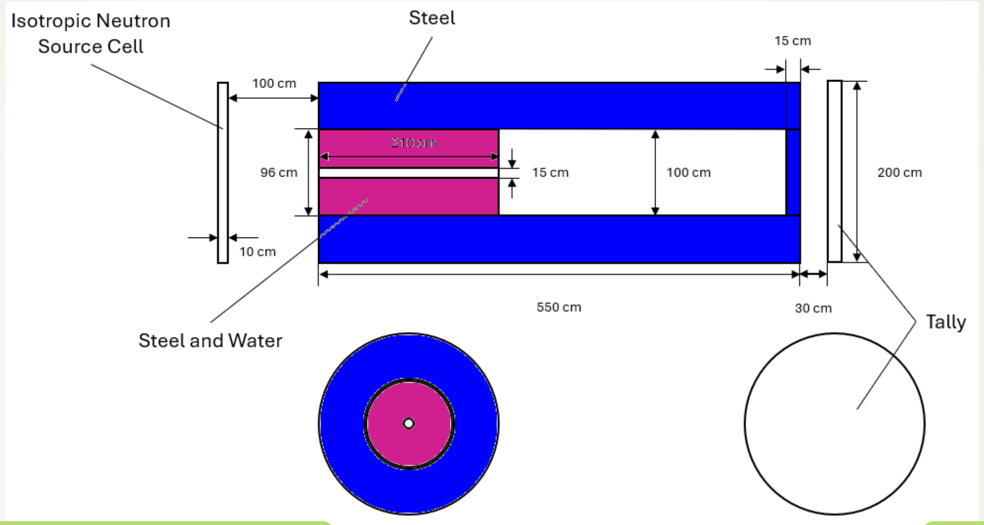
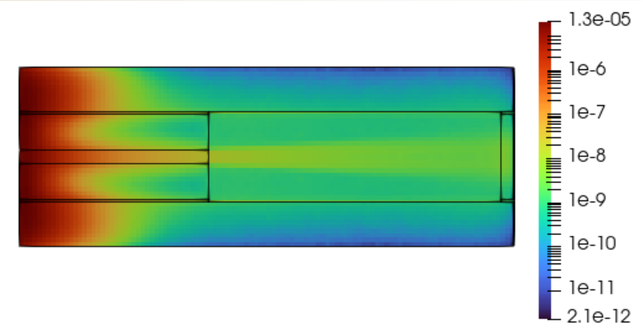
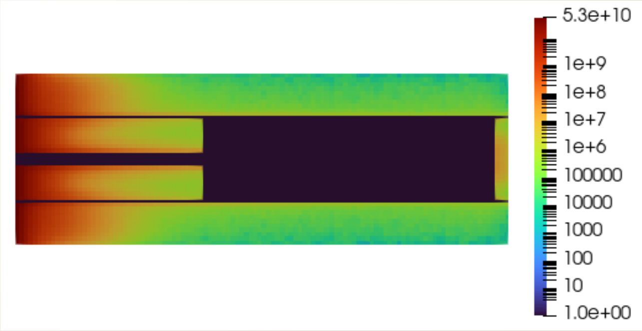


Neutron MC Simulation



Activation



R2S-UNED

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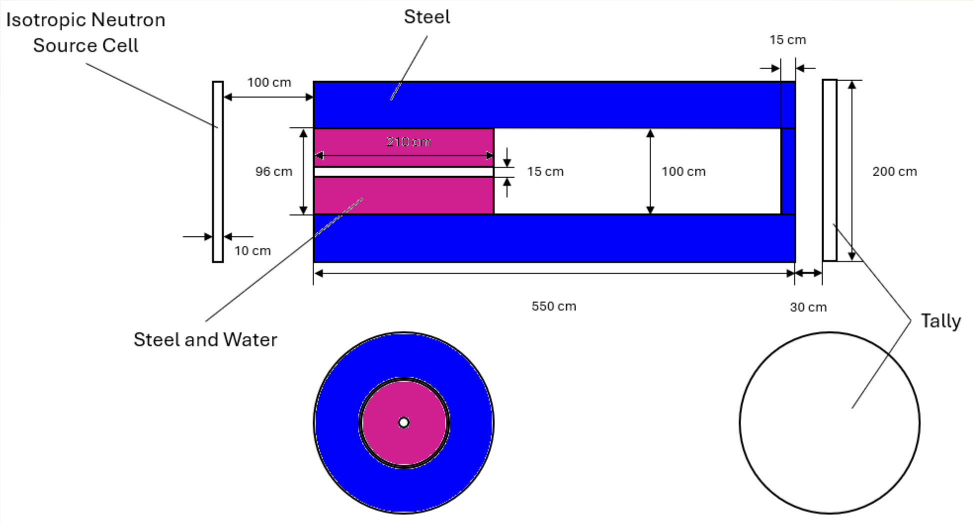
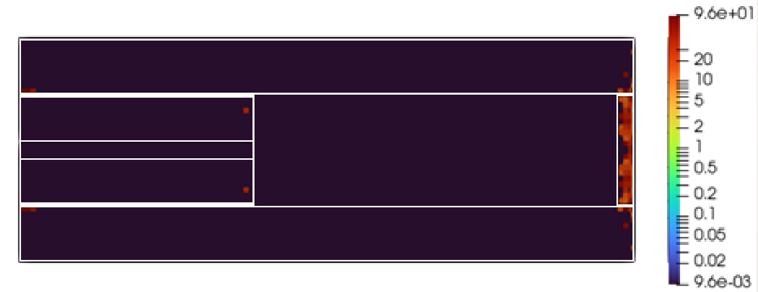


Sensitivity Coefficients



R2S-UNED

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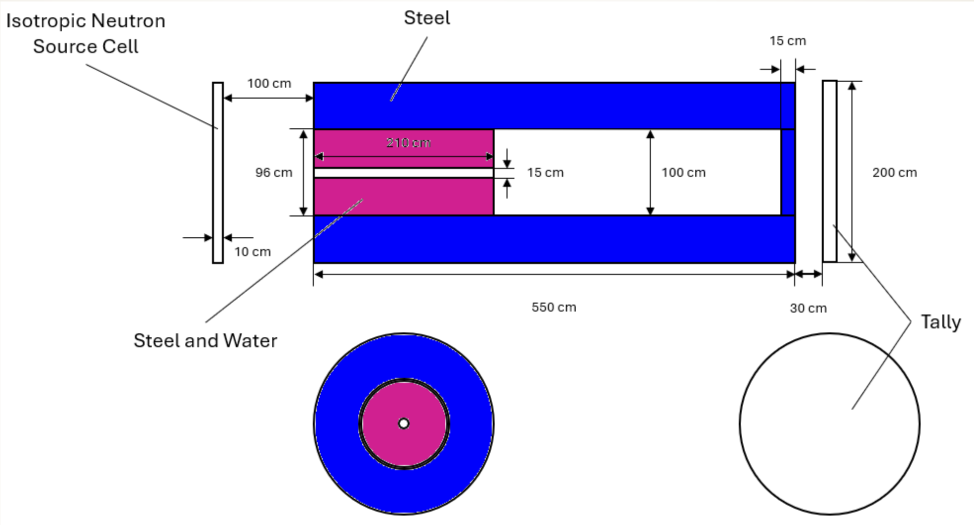

	SDR	F	$\sigma_{SDR_{MC1}}$	$\sigma_{SDR_{MC2}}$	σ_{SDR}
Brute Force	3.58×10^{-4}	–	–	4.96×10^{-6}	9.71×10^{-6}
Implicit Scheme	3.62×10^{-4}	3.54×10^{-4}	8.38×10^{-6}	4.96×10^{-6}	9.74×10^{-6}

DTE3 campaign at JET

criteria on MC1 reliability

OCT1, DD, IH1, 3H	Photon MC Simulation	
SDR (μSv/h)	Relative Stochastic Uncertainty	
$1.27 \times 10^{+1}$	0.0089	
Stochastic Uncertainty MC Simulation (Analogous Neutron MC Simulation)		
Random Variable (F) (μSv/h)	Relative Stochastic Uncertainty	Upper Limit
$1.26 \times 10^{+1}$	0.0004	0.0011
Complete R2S-UNED		
SDR (μSv/h)	Relative Stochastic Uncertainty	
$1.27 \times 10^{+1}$	0.0089	

Conclusions

- Methodology to evaluate overall stochastic uncertainties in two-step MC simulations.
- Computational requirements affordable.
- Criteria on MC1 reliability.

Future Works

- Map of stochastic uncertainties.
- Evaluation of systematic uncertainties.

Thanks for your attention,
if you have any questions…?